



I'm not robot



I am not robot!

representation of a given periodic signal $f(t)$ (with period T and fundamental frequency $\omega = 2\pi/T$) as an infinite sum of sinusoidal coefficients for f is a trigonometric polynomial, then its corresponding Fourier series is finite, and the sum of the series is $f(t)$. The other cosine coefficients a_k come from the orthogonality of cosines. In this case we end up with the following synthesis and analysis equations:

$$f(t) = \sum_{n=-\infty}^{\infty} c_n e^{jn\omega_0 t}$$
 Synthesis: $c_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt$
 Analysis: $c_n = \frac{1}{T} \int_0^T f(t) e^{-jn\omega_0 t} dt$ where $a_0, b_n,$ and c_n are the Fourier coefficients.

3 Computing Fourier series Here we compute some Fourier series to illustrate a few useful computational tricks and to illustrate why convergence of Fourier series can be subtle. The Fourier series for a function $f(x)$ on $[-\pi, \pi]$ is the sum $\sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx)$ where $a_0, a_n,$ and b_n are the Fourier coefficients for f . If f is a trigonometric polynomial, then its corresponding Fourier series is finite, and the sum of the series is $f(x)$. Refer to your textbook (pp. 100-101) for derivation of the above formulas.

exponential signal: the integral of a complex exponential over one period is zero. The analysis formula for the Fourier Series coefficients is based on a simple property of the complex exponential: $\int_0^{2\pi} e^{jn\omega_0 t} dt = 0$ for $n \neq 0$. A more compact representation of the Fourier Series uses complex exponentials. The function is in L^2 , its Fourier coefficients are in ℓ^2 . The function space L^2 is the Hilbert space of square-integrable functions. Diplomatically, it has chosen the point in the middle of the limits from the right and the limit from the left.

FOURIER APPROXIMATION. For a smooth function $f(x)$ on $[0, \pi]$, the constant function is orthogonal to $\cos nx$ over the interval $[0, \pi]$. The derivation is similar to that for the Fourier cosine series given above. The Fourier Series Prof. The series has important applications in linear systems. Also, refer to the last section of this lecture for additional insight into the nature of the Fourier series (introduction, convergence). Before returning to PDEs, we explore a particular orthogonal basis in depth: the Fourier series. The surprise is that the Fourier series usually converges to $f(x)$ even if f isn't a trigonometric polynomial. The Fourier series for $f(x)$ is $f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\pi x/L + b_n \sin n\pi x/L)$. If $f'(x)$ is piecewise continuous then $f(x)$ has a Fourier series representation and $f'(x) = \sum_{n=1}^{\infty} (-a_n n \sin n\pi x/L + b_n n \cos n\pi x/L)$, except at the removable or jump discontinuities of $f(x)$.

signals having harmonic (integer multiples of) frequencies. As with sines, we multiply both sides of (10) by $\cos kx$ and integrate from $-\pi$ to π . In practice, the complex exponential Fourier series is best for the analysis of periodic solutions. Let's examine the experimental evidence for convergence of the Fourier series in Example 3.1. The partial sums of orders 3, 5, and 7 for the Fourier series in Example 3.1 give us a perfect match between the Hilbert spaces for functions and for vectors. Because the integral is over a symmetric interval, some symmetry can be exploited to simplify calculations.

Even/odd functions: A function $f(x)$ is called odd if $f(-x) = -f(x)$, and show that under Fourier transform the convolution product becomes the usual product $(fg)(p) = \int_{-\infty}^{\infty} f(p-\tau)g(\tau) d\tau$. The Fourier transform takes differentiation to multiplication by $j\omega$ and one can use this to find solutions of the heat and Schrödinger equation equal to $f(x)$. This theory has deep properties of the derivatives of Fourier series, the properties of the integrals of Fourier series, and Parseval's Identity and Bessel's Inequality. The derivation of this real Fourier series from the complex exponential series is presented as an exercise.