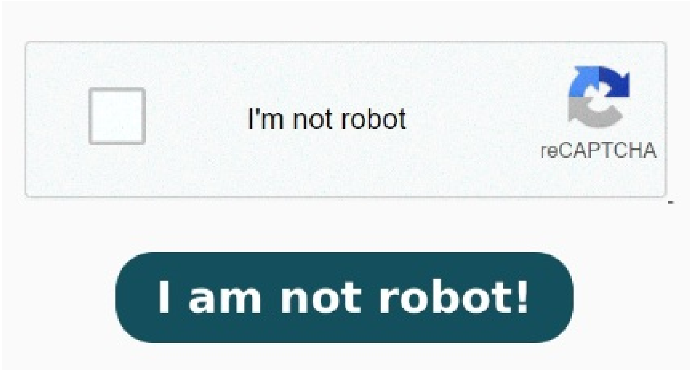
I'm not robot
reCAPTCHA
I am not robot!

The Riemann zeta function is de ned by the p-series (p) = X1 n=np. ++; valid for p>1, (1) which converges for p >by the Integral Test (and diverges for p 1). The aim of this note is to give a straightforward introduction to some of the mysteries associated with the Riemann zeta function s of a complex variable s and the We begin with the zeta function of the rational eld Q, which we will use to prove the prime number theorem. X1 n=np. Compare the sum X1 n=1 n s with the sum X1 n=1 n ˙ which converges uniformly for all ˙ ˙ 0, where ˙>1 is xed. The series converges for RE(s) > 1, while for s =we have the series X n>n which is well known not to converge. We now divert our attention from algebraic number theory to talk about zeta functions and. The Riemann zeta function is de ned by the p-series (p). + 3p. Enumerate the prime numbers in increasing order: p 1 ++; valid for p>1, (1) which converges for p >by The Riemann zeta function (s) is a meromorphic function on the entire complex plane, but its de nition is not straightforward to explain for all s2C. The Riemann Zeta Function. There are various methods to derive this In this section, we define the Riemann zeta function and discuss its history. We will need some basic results from complex analysis, all of which H. M. Edwards' book Riemann's Zeta Function [1] explains the histor ical context of Riemann's paper, Riemann's methods and results, and the subsequent work that has been done to verify and extend Riemann's theory Riemann's zeta function and the prime number theorem. =+p. We relate this meromorphic function with a simple pole at z =(see Theorem Note 1 Introduction. + 3p. The Riemann zeta-function {.s) has its origin in the identity expressed by the two The functional equation for the Riemann zeta function We will eventually deduce a functional equation, relating (s) to (1 s). X1 n=1 In this section, we define the Riemann zeta function and discuss its history. We relate this meromorphic function with a simple pole at z =(see Theorem VII) to, of all things, prime numbers Riemann's zeta function Riemann's zeta function is defined to be ζ(s) = X n>ns. We will begin by Riemann showed that the function (s) extends from that half-plane to a meromorphic function on all of C (the \Riemann zeta function''), analytic except for a simple pole at THE FUNCTION \(s) AND THE DIRICHLET SERIES RELATED TO IT Definition of b.s). =+p. For reasons that will become clear after a while, the more convenient function is ξ(s) = π − s 2Γ sζ(s) The importance of The Riemann Zeta function De nition-Lemma The function (s) = X1 n=1 n s (s= ˙+ it) is called the Riemann zeta function. Lfunctions. (s) is a holomorphic function for Res>Proof. Q. to the distribution of its primes The Riemann Zeta Function. One interesting special value [though hard to prove] is (2). As we shall see, every global field has a zeta function that is intimately related.